WESLEY COLLEGE
By daring & by doing

SOLUTIONS   Marks: /34

Calculator Allowed

1.   [5 marks]

   a) Explain what is meant by a simple random sample.

   A sample such that all possible samples of required size have an equal chance of being selected.

[1]

   b) Explain briefly how you could use your ClassPad to select a simple random sample of size 12 from a list of the 70 junior members of a tennis club.

   Number the members from 1 – 70.
   RandList (12, 1, 70)

[2]

   c) Give an example of a situation in which you might choose to take a stratified sample and explain why.

   Own choice + explanation

[2]

2.   [4 marks]

A group of would-be card sharks are investigating the results of dealing a card from a well-shuffled pack and checking its suit. Each card shark dealt a card 50 times, replacing and shuffling the cards before dealing the next one. They counted the number of times the card was a diamond and recorded the proportion of times out of 50.

   a) State the parameters for $\hat{p}$.

$$p = 0.25; \ (1 - p) = 0.75; n = 50$$

[2]

   b) What would be the mean and standard deviation of these results?

$$E(\hat{p}) = p = 0.25; \qquad SD(\hat{p}) = \sqrt{\frac{0.25 \times 0.75}{50}} = 0.061$$

[2]

3.   [5 marks]

As part of a Biology practical testing germination rates, each student saturated and placed 300 seeds in an incubator. When Mitch checked his incubator 5 days later, he found that 250 had germinated. Dr van Lieshout asks the class to calculate a 95% confidence interval based on their findings.

a) Find the confidence interval that should be obtained by Mitch.

$$\hat{p} = \frac{250}{300} = \frac{5}{6}$$

$$CI = \frac{5}{6} \pm 1.96 \times \sqrt{\frac{\frac{5}{6} \times \frac{1}{6}}{300}} = [0.7912, 0.8755]$$

[3]

Assuming that all the students calculate their confidence interval correctly,

b) State the proportion of the students you would expect to have a confidence interval that includes the population proportion of the distribution.

$$95\% = \frac{19}{20}$$

[1]

c) Explain why the probability of any one student's confidence interval including the population proportion is not 0.95.

The confidence interval either does, or does not, include the true population proportion ∴ probability is 0 or 1.

[1]

4.   [5 marks]

52% of Australian students travel to school by car. What is the probability that between 25 and 40 students from a sample of 80 students travel by car?

Check $\begin{matrix} np = 80 \times 0.52 = 41.6 \\ n(1-p) = 80 \times 0.48 = 38.4 \end{matrix} \Big\} > 10$ *or that sample size* > 30 *so can use the normal distribution as a model.*
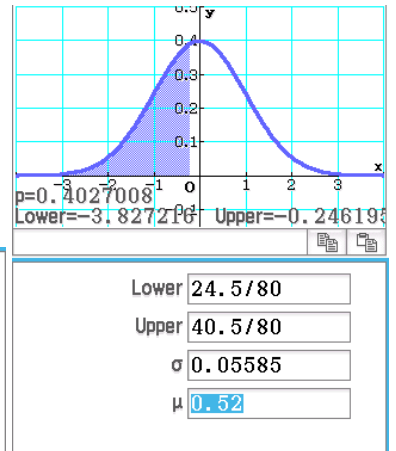
$$X \sim N(0.52, 0.05585^2)$$

prob	0.4027008
z Low	−3.827216
z Up	−0.246195
σ	0.05585
μ	0.52

Lower	24.5/80
Upper	40.5/80
σ	0.05585
μ	0.52

$$SD = \sqrt{\frac{p(1-p)}{n}} = 0.05585$$

$$P\left(\frac{24.5}{80} < X < \frac{40.5}{80}\right) = 0.4027$$

[5]

5.   [6 marks]

The number of customers, x, waiting in Nick's barber's shop in Angelo St is defined by this probability distribution:

x	0	1	2	3	4	5
P(x)	0.2	0.3	0.3	0.1	0.04	0.06

a) Determine the mean and the standard deviation of this distribution.

$$mean = 1.66 \qquad\qquad SD = 1.336$$

[2]

Mr Purdue went to get a haircut from Nick on 10 occasions last year and the average number of customers waiting was 1.9.

b) Is he correct in assuming this sample proportion is part of a normal distribution? Justify your answer.

Not really as sample size is too small. (Should be ≥ 30)

[1]

c) Use an appropriate probability model to determine whether, for a sample of 50 customers, an average of 1.9 customers or more waiting is consistent with the data calculated in part a).

$$Z \sim N(0, 1^2) \qquad Z = \frac{1.9 - 1.66}{1.336} = 0.1796$$

Yes, as it is 0.18σ above the mean.

[3]

6.   [6 marks]

A cinema advertising company employed a market research team to measure the effectiveness of the advertisements shown during film sessions.

The first 100 film-goers who left the 9pm session at a cinema complex were asked to recall the advertisements that they had seen. Of these, 58 recalled at least one advertisement.

a) Using the information above, calculate an approximate 90% confidence interval for the true proportion of film-goers who recalled at least one advertisement. Round your answer to 2 decimal places.

$$\hat{p} = \frac{58}{100} = 0.58$$

$$CI = 0.58 \pm 1.645 \times \sqrt{\frac{0.58 \times 0.42}{100}} = [0.4988, 0.6612] = [0.50, 0.66]$$

[3]

From the information collected, the marketing research team claimed that *'more than half'* of the film-goers recall cinema advertisements.

b) Given the sampling process and the confidence interval you have calculated; do you think that this claim is *fair*? Give *two* reasons for your answer.
    NO, because:
        ✓ *The selection process is not truly random. It consisted of the first people to leave a single, late evening session and so this group may be unrepresentative of all film-goers.*
        ✓ *While 50% lies near the lower end of the CI, it is possible that less than 50% can recall an advertisement. So strictly speaking the market research claim is not justified.*

[3]

7.   [3 marks]

The tourist information centre staff in Elizabeth Quay want to determine the proportion of people to within 2%, at a level of confidence of 99%, to work out whether they should request a new tourist advertisement campaign.
How many enquiries, to the nearest 50, would need to be noted at the information centre?

Use $\hat{p} = 0.5$ $n = \frac{z^2 \hat{p}(1-\hat{p})}{E^2} = \frac{2.576^2 \times 0.5 \times 0.5}{0.02^2} = 4147.36$

or $n = \frac{z^2 \hat{p}(1-\hat{p})}{E^2} = \frac{2.576^2 \times 0.5 \times 0.5}{0.01^2} = 16589.44$

Would require either 4150 or 16 600 enquiries

[3]